Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

2216 – 7304

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Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

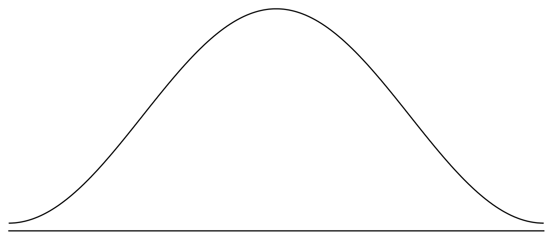
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A random variable X is distributed normally with a mean of 20 and standard deviation of 4.

(a) On the following diagram, shade the region representing $\mathrm{P}(X \le 25)$. [2]



(b) Write down $\mathrm{P}(X \le 25)$, correct to two decimal places. [2]

(c) Let $\mathrm{P}(X \le c) = 0.7$. Write down the value of c. [2]

2. [Maximum mark: 6]

Let $f(x) = x^2$ and $g(x) = 3\ln(x + 1)$, for $x > -1$.

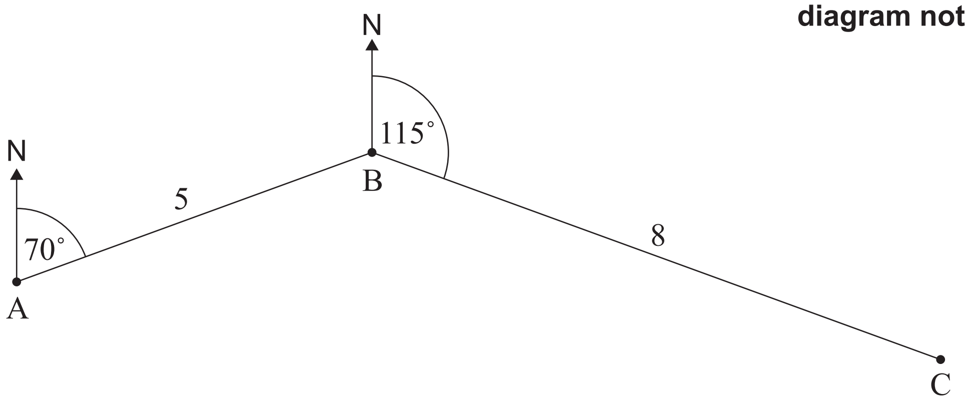
(a) Solve $f(x) = g(x)$. [3]

(b) Find the area of the region enclosed by the graphs of f and g. [3]

3. [Maximum mark: 7]

The following diagram shows three towns A, B and C. Town B is $5\,\text{km}$ from Town A, on a bearing of 070°. Town C is $8\,\text{km}$ from Town B, on a bearing of 115°.

diagram not to scale



(a) Find $A\hat{B}C$. [2]

(b) Find the distance from Town A to Town C. [3]

(c) Use the sine rule to find $A\hat{C}B$. [2]

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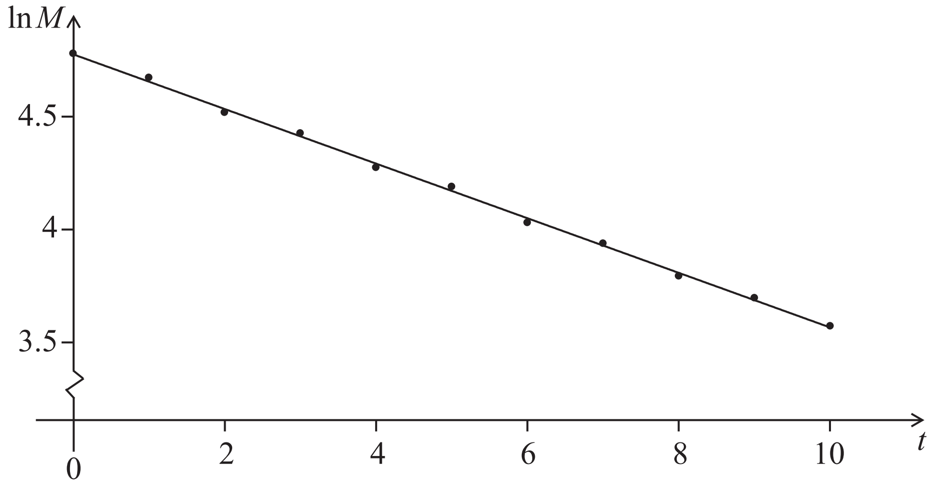
4. [Maximum mark: 6]

(a) Find the term in x^6 in the expansion of $(x+2)^9$. [4]

(b) Hence, find the term in x^7 in the expansion of $5x(x+2)^9$. [2]

5. [Maximum mark: 6]

The mass M of a decaying substance is measured at one minute intervals. The points $(t, \ln M)$ are plotted for $0 \le t \le 10$, where t is in minutes. The line of best fit is drawn. This is shown in the following diagram.



The correlation coefficient for this linear model is $r = -0.998$.

(a) State **two** words that describe the linear correlation between $\ln M$ and t. [2]

(b) The equation of the line of best fit is $\ln M = -0.12t + 4.67$. Given that $M = a \times b^t$, find the value of b. [4]

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6. [Maximum mark: 6]

In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

7. [Maximum mark: 8]

Note: One decade is 10 years

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

(a) (i) Find the value of k.

(ii) Interpret the meaning of the value of k. [3]

(b) Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. [5]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A factory has two machines, A and B. The number of breakdowns of each machine is independent from day to day.

Let A be the number of breakdowns of Machine A on any given day. The probability distribution for A can be modelled by the following table.

a	0	1	2	3
$\mathrm{P}(A=a)$	0.55	0.3	0.1	k

(a) Find k. [2]

(b) (i) A day is chosen at random. Write down the probability that Machine A has no breakdowns.

(ii) Five days are chosen at random. Find the probability that Machine A has no breakdowns on exactly four of these days. [3]

Let B be the number of breakdowns of Machine B on any given day. The probability distribution for B can be modelled by the following table.

b	0	1	2	3
$\mathrm{P}(B=b)$	0.7	0.2	0.08	0.02

(c) Find $\mathrm{E}(B)$. [2]

On Tuesday, the factory uses both Machine A and Machine B. The variables A and B are independent.

(d) (i) Find the probability that there are exactly two breakdowns on Tuesday.

(ii) Given that there are exactly two breakdowns on Tuesday, find the probability that both breakdowns are of Machine A. [8]

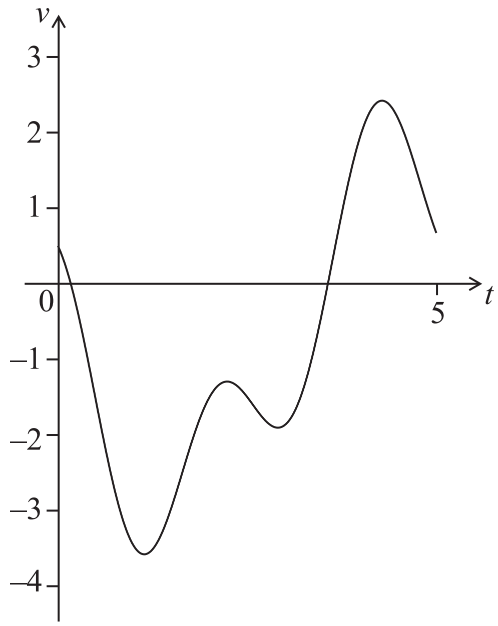
Do **not** write solutions on this page.

9. [Maximum mark: 14]

A particle P moves along a straight line so that its velocity, v ms^{-1}, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \le t \le 5$. The initial displacement of P from a fixed point O is 4 metres.

(a) Find the displacement of P from O after 5 seconds. [5]

The following sketch shows the graph of v.



(b) Find when P is first at rest. [2]

(c) Write down the number of times P changes direction. [2]

(d) Find the acceleration of P after 3 seconds. [2]

(e) Find the maximum speed of P. [3]

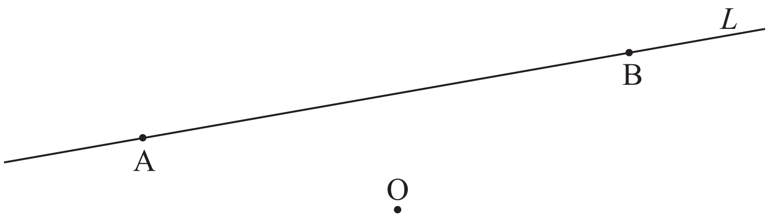
Do **not** write solutions on this page.

10. [Maximum mark: 16]

The points A and B lie on a line L, and have position vectors $\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$ respectively.

Let O be the origin. This is shown on the following diagram.

diagram not to scale



(a) Find $\overrightarrow{AB}$. [2]

The point C also lies on L, such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

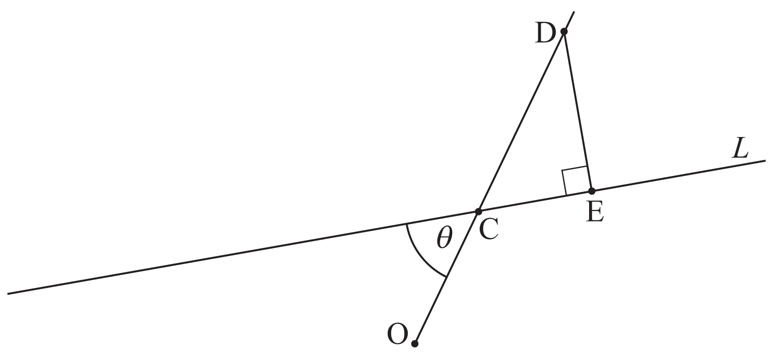
(b) Show that $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$. [3]

Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{OC}$.

(c) Find θ. [5]

Let D be a point such that $\overrightarrow{OD} = k\overrightarrow{OC}$, where $k > 1$. Let E be a point on L such that $\mathrm{C\hat{E}D}$ is a right angle. This is shown on the following diagram.

diagram not to scale



(d) (i) Show that $\left|\overrightarrow{DE}\right| = (k-1)\left|\overrightarrow{OC}\right|\sin\theta$.

(ii) The distance from D to line L is less than 3 units. Find the possible values of k. [6]